

Designing Geometric Parameters of Axisymmetrical Cassegrain Antenna and Corrugated Horn Feed for Circularly Polarized Communications

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ABSTRACT

This paper presents steps in designing dimensions of antenna and feed in axisymmetrical Cassegrain Antennas. Corrugated conical horns are used as feeds in applications with circular polarization. In the initial design steps, diameter of the main reflector is determined considering communication link budget, then feed dimensions are designed to create optimum aperture efficiency. In the last step of design, considering feed dimensions and the amount of its center phase displacements with respect to its aperture plane, dimensions of subreflector is determined to avoid additional blockage. To illustrate the procedure, an applicative design example is presented in X-band.

Keywords

Dual reflector antennas, Cassegrain antenna, main reflector, subreflector, blockage efficiency, illumination efficiency, corrugated conical horn.

1. INTRODUCTION

High gain aperture antennas with pencil beam pattern have vast applications in point to point communications, radars, tracking and exact positioning systems. Among aperture antennas, some applications like mono pulse tracking systems use dual reflector antennas due to their advantages [1]. Between dual reflector antennas, Cassegrain antenna has been found more useful. It has high effective F/D (focal distance to diameter) ratio. Also it has little sensitivity to cross polarization components, and simpler possibility of feed adjustment, since the antenna feed is placed near its main reflector apex. In some satellite ground stations for receiving picture signals with the ability to track satellite orbiting around the earth, usually a central corrugated conical horn feed is used for data reception. Four other lateral feeds are located around the main feed, to create sum and difference pattern and make necessary control signals to track the satellite in horizontal and vertical positions [2]. In designing such antennas a series of electrical and mechanical compromises must be considered. For example increasing antenna diameter will improve its gain but will make it heavier and will complicate the design of the holding and supporting structures. In this paper the process of designing

the feed and antenna are described considering these limitations. In the second and third part of the paper, geometric structure of antenna and feed are described respectively. In the forth part, design procedure is explained along with a design example which clarifies the design procedure much more.

2. Geometric Structure and related design equations in Cassegrain antennas

Fig. 1 shows geometric structure of Cassegrain antenna. This structure uses a parabolic surface as main reflector and a hyperbolic one as subreflector. The focal point of paraboloid and first focal point of hyperboloid are located on each other and the antenna feed phase center is located in the second focal point of the hyperbolic surface. Geometric parameters in the figure are defined as follows:

D : the main reflector diameter.

D_s : the subreflector diameter.

F : focal distance of the main reflector.

$2C$: distance between focal points of subreflector.

ψ_0 : subtended angle of the main reflector.

θ_0 : subtended angle of the subreflector.

F_e : Effective focal distance of the structure.

in Fig. 1, the right parabolic surface shows the equivalent reflecting surface of the structure.

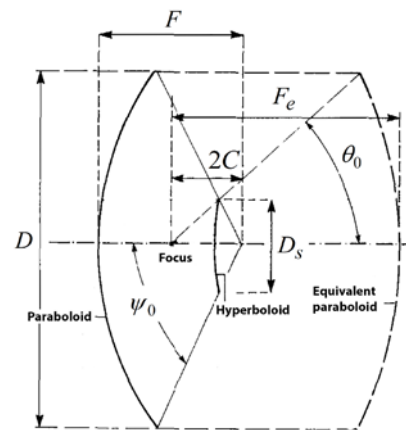


Fig. 1. Geometric structure of conventional Cassegrain antenna.

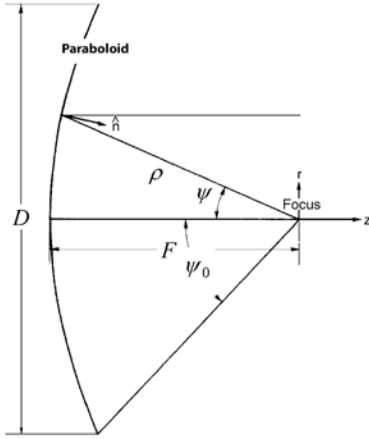


Fig. 2. Geometric structure of parabolic reflector.

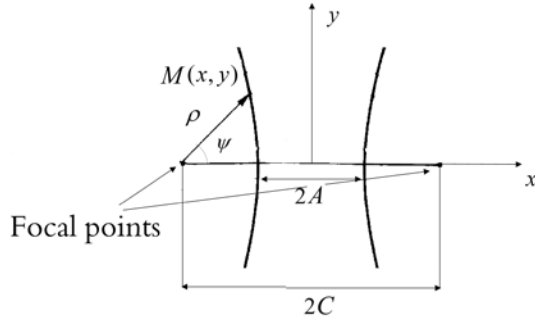


Fig.3. A set of hyperbolic surfaces.

Magnification factor is defined as

$$M = \frac{F_e}{F} \quad (1)$$

Eccentricity factor of subreflector, e , and M are related to each other as

$$e = \frac{M+1}{M-1} \quad (2)$$

Relation between e and θ_0 , ψ_0 is as following

$$e = \frac{\sin \frac{1}{2}(\psi_0 + \theta_0)}{\sin \frac{1}{2}(\psi_0 - \theta_0)} \quad (3)$$

There are four independent parameters, D, d, θ_0, ψ_0 , in describing this geometric structure.

Main reflector is determined by tow parameters, $D, F/D$. Fig. 2 shows a hyperbolic surface. The equations governing this structure are

$$\rho + \rho \cos \psi = 2F \Rightarrow \rho = \frac{F}{\cos^2(\psi/2)} \quad (4)$$

$$\sin \psi_0 = \frac{D}{2\rho} \quad (5)$$

We get from the above two relations

$$\tan\left(\frac{\psi_0}{2}\right) = \frac{D}{4F} \quad (6)$$

so ψ_0 can be determined from F/D ratio. Fig. 3 shows a set of hyperbolic surfaces, in which the distance between focal points is $2C$. If an arbitrary point $M(x, y)$ is considered on this surface, x, y will satisfy the following

$$\left(\frac{x}{A}\right)^2 - \left(\frac{y}{B}\right)^2 = 1, \quad e = \frac{C}{A}, \quad A^2 + B^2 = C^2 \quad (7)$$

With A and C known, hyperbolic equation will be known too. In Fig. 3, ρ and ψ are related as

$$\rho = \frac{A(e^2 - 1)}{1 + e \cos(\psi)} \quad (8)$$

aperture antenna gain is obtained from [3]

$$G = \varepsilon_{ap} D_u = \varepsilon_{ap} \frac{4\pi}{\lambda^2} A_p \quad (9)$$

Where D_u is the antenna directivity, ε_{ap} is the aperture efficiency in the range $0 \leq \varepsilon_{ap} \leq 1$, λ is the wavelength in free space and A_p is physical aperture area of the antenna.

The gain is proportional with A_p . Aperture efficiency may be written as the product of subefficiencies as

$$\varepsilon_{ap} = \varepsilon_t \varepsilon_s \varepsilon_{blk} \quad (10)$$

In the above equation, ε_t is defined as the taper efficiency and ε_s is called spillover efficiency, ε_{blk} is blockage efficiency and is due to the blocking of the radiated field of reflector by the objects inserted in front of it. To avoid gain reduction caused by blockage, some applications use offset fed antennas. but in some applications like monopulse tracking systems in which four antennas around the main feed must provide exactly symmetric radiation pattern, offset fed antennas are rarely used and blockage caused by subreflector will be inevitable. In (10) losses due to holding struts, random surface tolerances, ohmic losses, gain losses due to cross polarization components, etc. are neglected. Aperture taper efficiency and spillover efficiency are obtained from the followings

$$\varepsilon_t = \frac{4F^2 \left| \int_0^{2\pi} \int_0^{\psi_0} F_f(\psi, \phi) \tan\left(\frac{\psi}{2}\right) d\psi d\phi \right|^2}{\pi^2 \int_0^{2\pi} \int_0^{\psi_0} |F_f(\psi, \phi)|^2 \sin(\psi) d\psi d\phi} \quad (11)$$

$$\varepsilon_s = \frac{\int_0^{2\pi} \int_0^{\psi_0} |F_f(\psi, \phi)|^2 \sin(\psi) d\psi d\phi}{\int_0^{2\pi} \int_0^{\pi} |F_f(\psi, \phi)|^2 \sin(\psi) d\psi d\phi} \quad (12)$$

In the above equations $F_f(\psi, \phi)$ is the electric field of antenna feed, supposing corrugated conical structure as antenna feed, one can estimate the field distribution approximately symmetrical with respect to ϕ and as following with respect to ψ

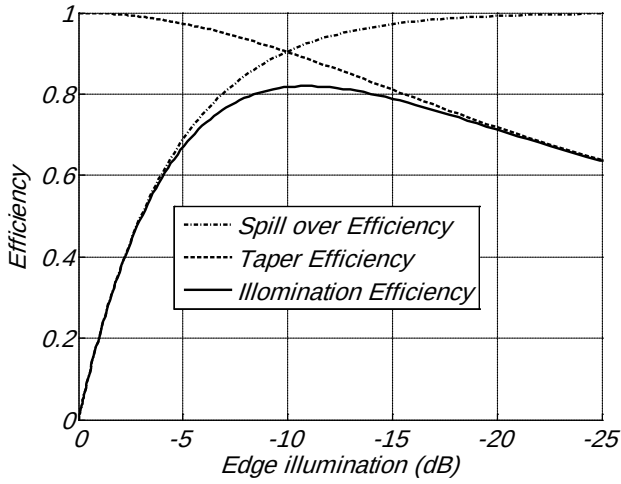


Fig. 4. Taper, spill over and illumination efficiencies with respect to edge illumination in circular aperture antennas.

$$F_f(\psi) = \begin{cases} \cos^2(\psi) & \psi \leq \frac{\pi}{2} \\ 0 & \psi > \frac{\pi}{2} \end{cases} \quad (13)$$

By the above assumptions, illumination efficiency, $\varepsilon_i = \varepsilon_t \varepsilon_s$, may be analytically obtained as

$$\varepsilon_i = 40 \cot^2\left(\frac{\psi_0}{2}\right) \times \left[\sin^4\left(\frac{\psi_0}{2}\right) + \ln\left(\cos\frac{\psi_0}{2}\right) \right]^2 \quad (14)$$

In Fig. 4, ε_i , ε_t , ε_s are plotted with respect to edge illumination (EI). EI is defined as follows

$$EI = \left(\frac{1 + \cos \psi_0}{2} \right) \cos^2 \psi_0 \quad (15)$$

equ. (15) is the product of obliquity factor and antenna pattern. Wave front emerging from the feed travel more distance when they hit to the edge of the reflector than when they hit to its apex, so field distribution at the antenna edge would be weaker than its apex. After reflection from the reflector, all rays are parallel and no longer suffer amplitude decay.

From Fig. 4, it is observed that maximum illumination efficiency ε_i is obtained when EI is almost -11 dB. In (10), blockage efficiency, ε_{blk} , is obtained from [4]

$$\varepsilon_{blk} = \left(1 - \frac{A_b}{\varepsilon_t A_p} \right)^2 \quad (16)$$

In (16), A_b is the blockage area which has blocked the A_p , aperture region, ε_t is aperture taper efficiency demonstrated by (11). For the case of maximum illumination efficiency, as can be observed from Fig. 4, $\varepsilon_t = 0.89$. It is seen from (16) that to increase ε_{blk} , A_b must be made as small as possible.

3. Geometric structure and design relations in corrugated conical horn feed

Fig. 5, shows the geometric structure of corrugated conical horn, which is used in applications with circular polarization

and has advantages over conventional conical horn feeds. By corrugating internal wall of conical horn, the phase centers of H and E plane and also feed pattern in H and E planes would be approximately the same, and cross polarization components reduce [5].

In designing dimensions of corrugated horn feeds, a parameter called S is defined as

$$S = \frac{a^2}{2\lambda R} \quad (17)$$

in (17), a is the radius of feed aperture, R is the slant radius and λ is the wavelength. Fig. 6, describes radiation pattern of corrugated conical horn with respect to $\frac{2\pi a}{\lambda} \sin \theta$ for various S parameters.

S parameters.

The phase center of the radiated waves is not in the aperture plane and is replaced toward its apex by L_p . The amount of displacement depends on the horn slant radius and S parameter and is tabulated in table 1.

In Fig.5, slot depth, d, must be in the range $\lambda/4 < d < \lambda/2$, till they have capacitive input impedance. The slot depth may be considered constant for narrow band applications.

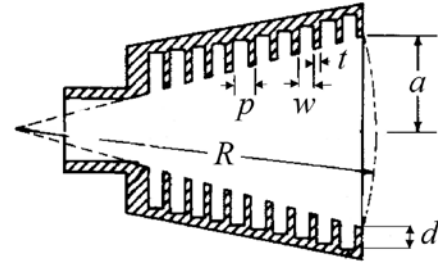


Fig.5. Geometric structure of corrugated conical horn.

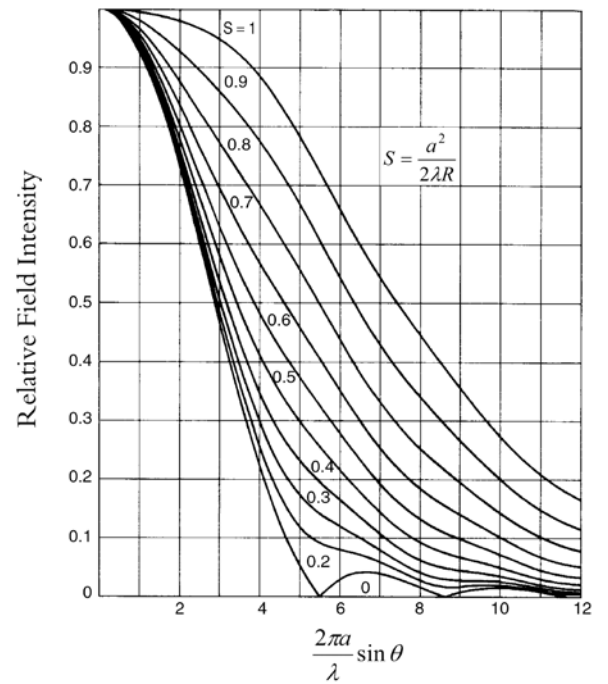


Fig.6. Radiation pattern of corrugated conical horn.

Table 1. Displacement amount of radiated waves phase center in corrugated horn behind its aperture plane for various S parameters.

S	L_p/R	S	L_p/R
0.00	0.0	0.36	0.386
0.04	0.005	0.40	0.464
0.08	0.020	0.44	0.542
0.12	0.045	0.48	0.614
0.16	0.080	0.52	0.673
0.20	0.124	0.56	0.718
0.24	0.178	0.60	0.753
0.28	0.240	0.64	0.783
0.32	0.310	0.68	0.811

For wideband applications and for better impedance matching of feed at the horn throat to circular input waveguide, or aperture plane to free space, the slit depths may be variable and decreasing from $\lambda/2$ at the horn throat to $\lambda/4$ at the horn aperture. The depth of the first slit in the vicinity of horn aperture may be considered a few larger than $\lambda/4$ and is obtained by [6],

$$d = \frac{\lambda}{4} \exp\left(\frac{1}{2.5ka}\right) \quad (18)$$

where $k = \frac{2\pi}{\lambda}$,

other parameters which are indicated in Fig. 5, are w , t , p . choosing these parameters somehow depends on mechanical fabrication limitations and the amount of physical strength on the horn. But at least 4 slits per wavelength are necessary for reduction of cross polarization components of the radiated waves. These parameters are usually chosen in the range below [7]

$$t/w \approx 0.5, \quad p/\lambda < 0.2 \quad (19)$$

The horn slant angle has direct impact on horn bandwidth. For narrow band horns, this angle usually is taken in the range 10 degrees to 15 degrees to avoid long and heavy horn feeds. In horns with wideband applications the slant angle must not exceed 5 degrees.

At the horn throat, discontinuity between smooth wall circular waveguide and corrugated conical section, may cause the higher order modes to be excited. To reduce them, it would be better to start the throat section with smooth wall conical section as is shown in Fig. 5.

4. Procedures of designing physical dimensions of antenna and feed and design example

In this section, considering presented equations in the previous sections, we develop a step by step method in designing the structure and present a design example later.

Step 1: determination of main reflector physical parameters, D , F , M , F_e .

Antenna diameter, D , is calculated by aiding from the communication link budget, but choosing F depends on some

mechanical or electrical limitations. Taking a large F/D ratio may result in complicating the design of subreflector holding structures and may increase blockage losses, also necessitated using large gain and heavier feeds in order to illuminate the subreflector from longer distance with proper pattern. On the other hand, reducing F/D ratio too much, may cause to increase the antenna sensitivity to the cross polarized components of the feed pattern. So F/D is usually taken in the range below

$$0.35 \leq F/D \leq 0.5 \quad (20)$$

Antenna magnification is usually chosen equal to 5.

Step 2: getting feed parameters, θ_0 , ψ_0 , a , R , t , w , p , d , L_p .

By knowing F_e from previous step, using (6), subtended angles of the subreflector and main reflector, θ_0 , ψ_0 , could be calculated. Then using (15) we obtain proper amount of relative electric field of feed, required to produce EI=-11 dB for maximum efficiency. then using Fig. 6 and choosing S parameter arbitrarily, the factor $\frac{2\pi a}{\lambda} \sin \theta$ would be obtained and from it, we can get the radius of feed aperture by knowing θ_0 , λ .

Slant radius of horn, R , is gotten from (17) knowing S , a , λ , and slit depth, d , is obtained from (18). From (19) we have $t/w = 0.5$, $p/\lambda = 0.2$. After determination of R and S , we get L_p from Table 1.

Step 3 – Subreflector parameters, D_s , C , A , estimation.

After determination of feed dimensions, in order to reduce blockage effects, subreflector diameter must be determined in such a way that shadow made by the feed on the reflector equals to the subreflector dimension. Considering Fig. 7, blockage angle of feed, α , may be determined by

$$\alpha = \tan^{-1}\left(\frac{a_f}{(2C - L_p)}\right) \quad (21)$$

Where $a_f = a + d$ and $C = eA$

Considering Fig. 7, and from (6) we get

$$D_{fb} = 4F \tan\left(\frac{\alpha}{2}\right) \quad (22)$$

and from (8) we have

$$D_s = 2 \frac{A(e^2 - 1) \sin(\psi_0)}{1 + e \cos(\psi_0)} \quad (23)$$

in eqs. (21) to (23) all parameters are known from previous steps except for A , and by setting $D_s = D_{fb}$, in order to reduce subreflector blockage, this parameter would be determined also. In order to get A we may create an error function as (24) and optimizing this function with MATLAB software [8] will result in the optimum value for A .

$$err(A) = \left| 4F \tan \left(\frac{\tan^{-1} \left(\frac{a_f}{(2Ae - L_p)} \right)}{2} \right) - 2 \frac{A(e^2 - 1) \sin(\psi_0)}{(1 + e \cos(\psi_0))} \right| \quad (24)$$

After finding A, C and D_s could be found too.

The steps in finding physical dimensions of the structure is finished in here and we continue to illustrate the presented method further by explaining a design example

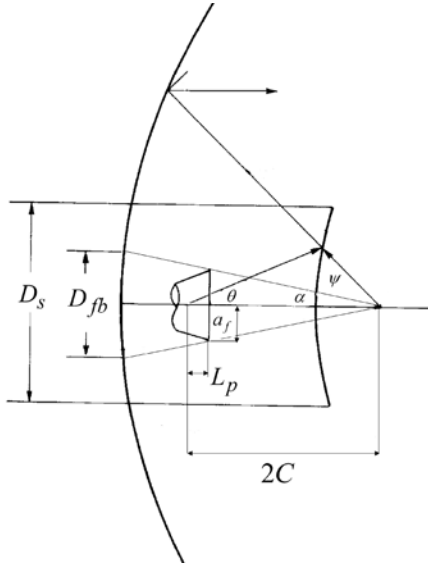


Fig.7. Aperture blockage in Cassegrain system.

Design example:

Design of antenna and feed of a satellite ground station for receiving image information with circular polarization is required. The antenna tracks satellite by mono pulse tracking method and its frequency of work is 8.11 GHz.

To solve the problem, by following the aforementioned steps we get the optimum dimensions of the structure.

Step 1:

Antenna diameter considering link budget determines as $D = 8.2(m)$.

We choose $F/D = 0.5$, $M = 5$.

Step 2:

$$F_e/D = M(F/D) = 5 \times 0.5 = 2.5$$

$$\theta_0 = 2 \tan^{-1} \left(\frac{1}{4F_e/D} \right) = 2 \tan^{-1} \left(\frac{1}{4 \times 2.5} \right) = 11.4^\circ$$

$$\psi_0 = 2 \tan^{-1} \left(\frac{1}{4F/D} \right) = 2 \tan^{-1} \left(\frac{1}{4 \times 0.5} \right) = 53.1^\circ$$

We choose S parameter equal to 0.2 arbitrarily.

From (15) for $EI = -11dB = 0.2818$ with $\theta_0 = 11.4^\circ$ for equivalent paraboloid, considering obliquity factor, the required relative electric field intensity at the edges must be

0.2439. By observing Fig.6, $\frac{2\pi a}{\lambda} \sin \theta$ equals to about 3.7,

$$\frac{2\pi a}{\lambda} \sin \theta = 3.7 \Rightarrow a = 11.02 \text{ cm}$$

from (17) the feed slant radius is calculated from S as

$$S = \frac{a^2}{2\lambda R} \Rightarrow R = \frac{a^2}{2\lambda S} = \frac{11.02^2}{2 \times 3.7 \times 0.2} = 82 \text{ cm}$$

considering (18), the output slit depth would be

$$d = \frac{\lambda}{4} \exp \left(\frac{1}{2.5ka} \right) = \frac{3.7}{4} \exp \left(\frac{1}{2.5 \times 18.6} \right) = 0.9 \text{ cm} \cong 1 \text{ cm}$$

and (19) earns

$$p = 0.2 \times 3.7 = 0.74 \text{ (cm)}, \quad t = w = 0.37 \text{ (cm)}$$

also from Table 1

$$\frac{L_p}{R} = 0.124 \Rightarrow L_p = 0.124 \times 82 = 10.2 \text{ (cm)}$$

Step 3-

in (24), $a_f = 12.02$ and from optimization of this function the

optimum value of A equals to 57.5 (cm), so

$$C = eA = 1.5 \times 57.5 = 86.25 \text{ (cm)}$$

and from (22) or (23) we get

$$D_s = D_{fb} = 60.5 \text{ (cm)}$$

The calculated physical parameters for this design example are summarized in Table 2 for easier reference.

Table 2- summary of the calculated results of the design example.

Parameter	Value	Parameter	Value
D	8.2 (m)	C	86.25 (cm)
F	4.1 (m)	a	11.02 (cm)
F_e	20.5 (m)	d	1 (cm)
D_s	60.5(cm)	t	0.37 (cm)
θ_0	11.4 °	w	0.37 (cm)
ψ_0	53.1 °	p	0.74 (cm)
A	57.5(cm)	L_p	10.2 (cm)

5. Conclusion

In this paper a method is presented for determination of geometric parameters of Cassegrain antenna and its feed. This method uses corrugated conical horn for receiving signals with circular polarization. Considering the compromises that exists between electrical and mechanical requirements, and using communication link budget, the diameter of the main reflector is obtained first, then taking into account required amount of edge field for obtaining highest efficiency, the feed dimensions are accounted, and center phase displacement of

radiated waves with respect to aperture plane of the feed is acquired. Then in order to minimizing the effect of central blockage, the size of feed shadow on the reflector, is set equal to the subreflector dimension and thus canonical distance and focal point position of the subreflector are gained. A design example is included in the paper to further illustration of the design operation.

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