

Statistical Analysis and Model Validation of Gompertz Model on different Real Data Sets for Reliability Modelling

By

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ABSTRACT

A very basic problem in reliability modeling is to obtain information about the form of the population from which the sample is drawn. Goodness of fit test is employed on different real data sets to determine how well the observed sample data "fits" on proposed model for reliability analysis. In this paper, we obtain the tables and graphs of critical values of Kolmogorov-Smirnov (KS) test, and Q-Q test for Gompertz model with two unknown parameters. These plots are used to investigate whether an assumed model adequately fits a set of data and we present power comparison between Computation of MLE using Newton-Raphson method and p-value with its corresponding D-value obtaining by KS-Test Q-Q test for model validation to obtain feasible real data sets which are most suitable for parameter estimation of Gompertz model. For this analysis, we used different tools which are developed in R language and environment for model analysis, model validation and estimation of parameters using method of maximum likelihood.

Keywords

Gompertz model, probability density function (pdf), cumulative distribution function (cdf), model validation, quantile-quantile(Q-Q) test, goodness of fit test

1. INTRODUCTION

The Gompertz model plays an important role in modeling human mortality and fitting actuarial tables. Historically, the Gompertz model was first introduced by Gompertz [7]. Recently, many authors have contributed to the studies of statistical methodology and characterization of this model; for example, Read [18], Makany [15], Rao and Damaraju [17], Franses [5], Chen [3] and Wu and Lee [22]. Garg et al. [6] studied the properties of the Gompertz model and obtained the maximum likelihood (ML) estimates for the

parameters. Gordon [8] provided the ML estimation for the mixture of two Gompertz models.

In this paper, we investigate the statistical properties of two parameter Gompertz model and then we check the validity of this model on different real data sets by using modulus operandi which are easy to understand and implement, and are based on intuitive and graphical techniques such as Q-Q plot test, Kolmogorov-Smirnov (K-S) test and plots the graph of empirical distribution function and fitted distribution function. These plots are used to investigate whether an assumed model adequately fits a set of proposed data. We present power comparison between these data sets obtaining by K-S test and Q-Q test for model validation to obtain feasible real data sets which are most suitable for parameter estimation of Gompertz model.

2. MODEL ANALYSIS

The Cumulative distribution function of Gompertz model with two parameters is given by

$$F(x; \alpha, \theta) = 1 - \exp\left\{-\frac{\theta}{\alpha}(1 - \exp(x\alpha))\right\} ; \quad (2.1)$$

Where $(\alpha, \theta) > 0, 0 < x < \infty$

where $\alpha > 0$ is the shape and $\theta > 0$ is the scale parameter. The two-parameter Gompertz model will be denoted by Gompertz (α, θ) .

The probability density function is given by

$$f(x; \alpha, \theta) = \theta \exp(x\alpha) \exp\left\{-\frac{\theta}{\alpha}(1 - \exp(x\alpha))\right\} ; \quad (2.2)$$

where $(\alpha, \theta) > 0, 0 < x < \infty$

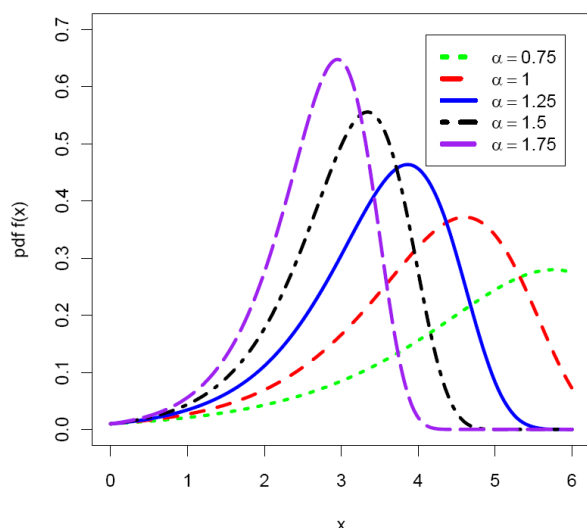


Fig 1 Plots of the probability density function of the Gompertz Model for $\theta=1$ and different values of α .

The R functions *dgompertz()* and *pgompertz()* can be used for the computation of pdf and cdf, respectively. Some of the typical Gompertz density functions for different values of α and for $\theta = 1$ are depicted in Figure 1. It is clear from the figure that the density function of the Gompertz model can take different shapes.

The reliability or survival function is

$$R(x; \alpha, \theta) = \exp\left\{\frac{\theta}{\alpha}\{1 - \exp(x\alpha)\}\right\} \quad (2.3)$$

The R function *sgompertz()* computes the reliability/survival function.

The hazard rate function is

$$h(x) = \theta \exp(x\alpha); \quad (\alpha, \theta) > 0 \quad (2.4)$$

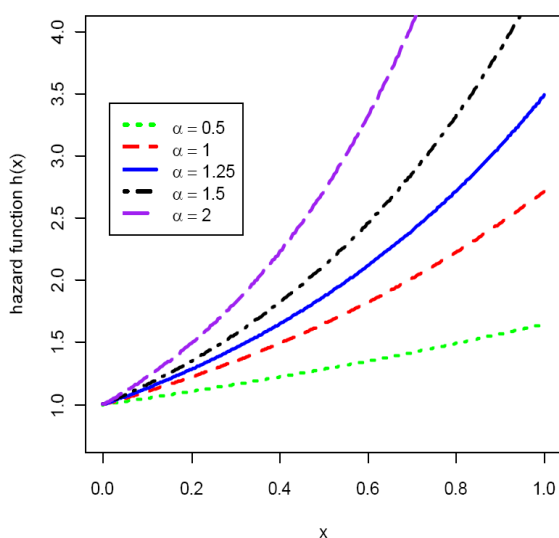


Fig. 2 Plots of the hazard function of the Gompertz Model for $\theta=1$ and different values of α .

The hazard rate is an increasing function. It has been graphed in Figure 2 for scale parameter $\theta = 1$ and different values of shape parameter α . The associated R function *hgompertz()* is used.

The cumulative hazard function $H(x)$ defined as

$$H(x) = -\log\{1 - F(x)\} \quad (2.5)$$

can be obtained with the help of *pgompertz()* function by choosing arguments *lower.tail=FALSE* and *log.p=TRUE*. i.e.

- pgompertz(x, alpha, theta, lower.tail=FALSE, log.p=TRUE)

Two other relevant functions useful in reliability analysis are failure rate average (fra) and conditional survival function(crf) The failure rate average of X is given by

$$FRA(x) = \frac{H(x)}{x} = \frac{\int_0^x h(x) dx}{x}, \quad x > 0, \quad (2.6)$$

where $H(x)$ is the cumulative hazard function. An analysis for $FRA(x)$ on x permits to obtain the IFRA and DFRA classes.

The survival function (s.f.) and the conditional survival of X are defined by

$$R(x) = 1 - F(x)$$

$$\text{and } R(x | t) = \frac{R(x+t)}{R(x)}, \quad t > 0, x > 0, R(\cdot) > 0, \quad (2.7)$$

respectively, where $F(\cdot)$ is the cdf of X . Similarly to $h(x)$ and $FRA(x)$, the distribution of X belongs to the new better than used (NBU), exponential, or new worse than used (NWU) classes, when $R(x | t) < R(x)$, $R(t | x) = R(x)$, or $R(x | t) > R(x)$, respectively.

The R functions *hra.gompertz()* and *crf.gompertz()* can be used for the computation of failure rate average (fra) and conditional survival function(crf), respectively.

The quantile function is given by

$$x_q = \theta \left\{ \frac{q}{1-q} \right\}^{1/\beta}; \quad 0 < q < 1. \quad (2.8)$$

The computation of quantiles the R function *qgompertz()* can be used.

Let U be the uniform (0,1) random variable and $F(\cdot)$ a cdf for which $F^{-1}(\cdot)$ exists. Then $F^{-1}(u)$ is a draw from distribution $F(\cdot)$. Therefore, the random deviate can be generated from Gompertz(α, θ) by

$$x = \theta \left\{ \frac{u}{1-u} \right\}^{1/\beta}; \quad 0 < u < 1. \quad (2.9)$$

where, u has the $U(0, 1)$ distribution. The R function `rgompertz()`, generates the random deviate from $Gompertz(\alpha, \theta)$.

2.1 Computation of MLE

In this section, we briefly discuss the MLE's of the two-parameter Gompertz model and discuss their asymptotic properties to obtain approximate confidence intervals based on MLE's [4].

Let $\underline{x} = (x_1, \dots, x_n)$ be a random sample of size n from $Gompertz(\alpha, \theta)$, then the log-likelihood function $L(\alpha, \theta)$ can be written as;

$$L(\alpha, \theta) = \theta^n e^{\alpha \sum x_i} \exp \left[\frac{\theta}{\alpha} \left(ne^{\alpha} - \sum e^{\alpha x_i} \right) \right] \quad (2.1.1)$$

Therefore, to obtain the MLE's of α and θ , we can maximize (2.1.1) directly with respect to α and θ or we can solve the following two non-linear equations using Newton-Raphson method

$$\frac{\partial \ln L}{\partial \alpha} = \sum_{i=1}^n x_i + \frac{\theta}{\alpha} \left[ne^{\alpha} - \sum_{i=1}^n x_i e^{\alpha x_i} \right] - \frac{\theta}{\alpha^2} \left[ne^{\alpha} - \sum_{i=1}^n e^{\alpha x_i} \right]$$

and

$$\frac{\partial \ln L}{\partial \theta} = \frac{n}{\theta} + \frac{1}{\alpha} \left[ne^{\alpha} - \sum_{i=1}^n e^{\alpha x_i} \right]$$

Let us denote the parameter vector by $\underline{\delta} = (\alpha, \theta)$

and the corresponding MLE of $\underline{\delta}$ as $\hat{\underline{\delta}} = (\hat{\alpha}, \hat{\theta})$ then the asymptotic normality results in

$$(\hat{\underline{\delta}} - \underline{\delta}) \rightarrow N_2 \left(\underline{0}, (I(\underline{\delta}))^{-1} \right) \quad (2.1.2)$$

where $I(\underline{\delta})$ is the Fisher's information matrix given by

$$I(\underline{\delta}) = - \begin{bmatrix} E \left(\frac{\partial^2 \ln L}{\partial \alpha^2} \right) & E \left(\frac{\partial^2 \ln L}{\partial \alpha \partial \theta} \right) \\ E \left(\frac{\partial^2 \ln L}{\partial \alpha \partial \theta} \right) & E \left(\frac{\partial^2 \ln L}{\partial \theta^2} \right) \end{bmatrix} \quad (2.1.3)$$

In practice, it is useless that the MLE has asymptotic variance $(I(\hat{\underline{\delta}}))^{-1}$ because we do not know $\underline{\delta}$. Hence, we approximate the asymptotic variance by "plugging in" the estimated value of the parameters. The common procedure is to use observed Fisher information matrix $O(\hat{\underline{\delta}})$ (as an estimate of the information matrix $I(\underline{\delta})$) given by

$$O(\hat{\underline{\delta}}) = - \begin{bmatrix} \frac{\partial^2 \ln L}{\partial \alpha^2} & \frac{\partial^2 \ln L}{\partial \alpha \partial \theta} \\ \frac{\partial^2 \ln L}{\partial \alpha \partial \theta} & \frac{\partial^2 \ln L}{\partial \theta^2} \end{bmatrix}_{(\hat{\alpha}, \hat{\theta})} = -H(\theta) \Big|_{\hat{\underline{\delta}}} \quad (2.1.4)$$

where H is the Hessian matrix, $\underline{\delta} = (\alpha, \theta)$ and $\hat{\underline{\delta}} = (\hat{\alpha}, \hat{\theta})$. The Newton-Raphson algorithm to maximize the likelihood produces the observed information matrix. Therefore, the variance-covariance matrix is given by

$$\left(-H(\theta) \Big|_{\hat{\underline{\delta}}} \right)^{-1} = \begin{pmatrix} \text{Var}(\hat{\alpha}) & \text{cov}(\hat{\alpha}, \hat{\theta}) \\ \text{cov}(\hat{\alpha}, \hat{\theta}) & \text{Var}(\hat{\theta}) \end{pmatrix} \quad (2.1.5)$$

Hence, from the asymptotic normality of MLEs, approximate $100(1-\gamma)\%$ confidence intervals for α and θ can be constructed as

$$\hat{\alpha} \pm z_{\gamma/2} \sqrt{\text{Var}(\hat{\alpha})} \quad \text{and} \quad \hat{\theta} \pm z_{\gamma/2} \sqrt{\text{Var}(\hat{\theta})}$$

where $z_{\gamma/2}$ is the upper percentile of standard normal variate.

2.2 Data Analysis

In this section we present six real data sets for illustration of the proposed methodology. These are

Data Set 1: SYS2.DAT - 86 time-between-failures [14] is converted to time to failures and scaled.

4.79, 7.45, 10.22, 15.76, 26.10, 28.59, 35.52, 41.49, 42.66, 44.36, 45.53, 58.27, 62.96, 74.70, 81.63, 100.71, 102.06, 104.83, 110.79, 118.36, 122.73, 145.03, 149.40, 152.80, 156.85, 162.20, 164.97, 168.60, 173.82, 179.95, 182.72, 195.72, 203.93, 206.06, 222.26, 238.27, 241.25, 249.99, 256.17, 282.57, 282.62, 284.11, 294.45, 318.86, 323.46, 329.11, 340.30, 344.67, 353.94, 398.56, 405.70, 407.51, 422.36, 429.93, 461.47, 482.62, 491.46, 511.83, 526.64, 532.23, 537.13, 543.06, 560.75, 561.60, 589.96, 592.09, 610.75, 615.65, 630.52, 673.74, 687.92, 698.15, 753.05, 768.25, 801.06, 828.22, 849.97, 885.02, 892.27, 911.90, 951.69, 962.59, 965.04, 976.98, 986.92, 1025.94

Data Set 2: The following data set includes the survival times (in days) of 72 guinea pigs infected with virulent tubercle bacilli, observed and reported by Bjerkedal [2].

10, 33, 44, 56, 59, 72, 74, 77, 92, 93, 96, 100, 100, 102, 105, 107, 107, 108, 108, 108, 109, 112, 113, 115, 116, 120, 121, 122, 122, 124, 130, 134, 136, 139, 144, 146, 153, 159, 160, 163, 163, 168, 171, 172, 176, 183, 195, 196, 197, 202, 213, 215, 216, 222, 230, 231, 240, 245, 251, 253, 254, 254, 278, 293, 327, 342, 347, 361, 402, 432, 458, 555

Data Set 3: The data is obtained from Lawless [13] and it represents the number of revolution before failure of each of 23 ball bearings in the life tests and they are as follows:

17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.80, 51.84, 51.96, 54.12, 55.56, 67.80, 68.44, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04, 173.40

Data Set 4: The data gives 100 observations on breaking stress of carbon fibres (in Gba) [16].

3.70, 2.74, 2.73, 2.50, 3.60, 3.11, 3.27, 2.87, 1.47, 3.11, 4.42, 2.41, 3.19, 3.22, 1.69, 3.28, 3.09, 1.87, 3.15, 4.90, 3.75, 2.43, 2.95, 2.97, 3.39, 2.96, 2.53, 2.67, 2.93, 3.22, 3.39, 2.81, 4.20, 3.33, 2.55, 3.31, 3.31, 2.85, 2.56, 3.56, 3.15, 2.35, 2.55, 2.59, 2.38, 2.81, 2.77, 2.17, 2.83, 1.92, 1.41, 3.68, 2.97, 1.36, 0.98, 2.76, 4.91, 3.68, 1.84, 1.59, 3.19, 1.57, 0.81, 5.56, 1.73, 1.59,

2.00, 1.22, 1.12, 1.71, 2.17, 1.17, 5.08, 2.48, 1.18, 3.51, 2.17, 1.69, 1.25, 4.38, 1.84, 0.39, 3.68, 2.48, 0.85, 1.61, 2.79, 4.70, 2.03, 1.80, 1.57, 1.08, 2.03, 1.61, 2.12, 1.89, 2.88, 2.82, 2.05, 3.65

Data Set 5: Aarset MV. How to identify bathtub hazard rate.
 IEEE Trans Reliability 1987;R-36(1):106 -108. (Failure time of 50 items [1].

0.1, 0.2, 1.0, 1.0, 1.0, 1.0, 2.0, 3.0, 6.0, 7.0, 11.0, 12.0, 18.0, 18.0, 18.0, 18.0, 21.0, 32.0, 36.0, 40.0, 45.0, 45.0, 47.0, 50.0, 55.0, 60.0, 63.0, 63.0, 67.0, 67.0, 67.0, 72.0, 75.0, 79.0, 82.0, 82.0, 83.0, 84.0, 84.0, 84.0, 85.0, 85.0, 85.0, 85.0, 86.0, 86.0

Data Set 6: The data is obtained from Lai et al. [12];
 Complete Data : Failure times of 20 components.

0.481, 1.196, 1.438, 1.797, 1.811, 1.831, 1.885, 2.104, 2.133, 2.144, 2.282, 2.322, 2.334, 2.341, 2.428, 2.447, 2.511, 2.593, 2.715, 3.218

2.3 Obtaining MLE on Proposed Data Sets

For obtaining the MLE(maximum likelihood estimation), we have started the iterative procedure by maximizing the log-likelihood function given in (2.1.1) directly with an initial guess for $\alpha = 0.01$ and $\theta = 0.05$ for away from the solution[11]. We have used *maxLik()* function in R with option Newton-Raphson method[19] and [21]. The iterative process stopped only after various no. of iterations depend on used data set[10]. The Table 1 shows the ML estimates and Log-Likelihood value of the parameters alpha and theta.

TABLE I. ML ESTIMATES WITH CORRESPONDING LOG-LIKELIHOOD

Data Set	MLE		Log-Likelihood
	alpha	theta	
1	0.001213069	0.001733294	-593.0077
2	0.004427568	0.002979251	-434.3901
3	0.01632877	0.00539636	-115.98
4	0.79109035	0.07691809	-149.125
5	0.020264029	0.009736145	-235.3266
6	1.96831467	0.01911059	-16.45955

3. MODEL VALIDATION

Most statistical methods assume an underlying model in the derivation of their results. However, when we presume that the data follow a specific model, we are making an assumption. If such a model does not hold, then the conclusions from such analysis may be invalid. Although hazard plotting and the other graphical methods can guide the choice of the parametric distribution, one cannot of course be

sure that the proper model has been selected. Hence model validation is still necessary to check whether we have achieved the goal of choosing the right model[17]. In this paper we outline some of the methods used to check model appropriateness.

3.1 Kolmogorov–Smirnov Test

The Kolmogorov–Smirnov test (K–S test) is a nonparametric test for the equality of continuous and that can be used to compare a sample with a reference probability model. The Kolmogorov–Smirnov statistic quantifies a distance between the empirical distribution function of the sample and the cumulative distribution function of the reference model [9].

- The Empirical Distribution Function(EDF)**

An estimate of $F(x) = P[X \leq x]$ is the proportion of sample points that fall in the interval $[-, x]$. This estimate is called the empirical distribution function(EDF). The EDF of an observed sample $x_1, x_2, \dots, x_n$ is defined by

$$F_n(x) = \begin{cases} 0 & \text{for } x < X_{1:n} \\ \frac{i}{n} & \text{for } X_{i:n} \leq x < X_{i+1:n}; i = 1, \dots, n-1 \\ 1 & \text{for } x \geq X_{n:n} \end{cases}$$

where $x_{1:n}, x_{2:n}, \dots, x_{n:n}$ is the ordered sample.

The Kolmogorov–Smirnov (K-S) test is a nonparametric *goodness-of-fit* test and is used to determine whether an underlying probability distribution ($F_n(x)$) differs from a hypothesized distribution ($F_0(x)$).

- Kolmogorov-Smirnov (K-S) distance**

The K-S distance between two distribution functions is defined as

$$D_n^+ = \max_{1 \leq i \leq n} |F_n(x) - F_0(x_i)|,$$

and

$$D_n^- = \max_{1 \leq i \leq n} |F_0(x_i) - F_n(x)|,$$

where $F_0(x_i)$ is the cumulative distribution function evaluated at x_i and $F_n(x)$ is the EDF. To perform the two-sided goodness of fit test $H_0 : F(x) = F_0(x)$ for all x , where F is a completely specified continuous distribution function against the alternative $H_1 : F(x) \neq F_0(x)$, for some x , the K-S statistic is

$$D_n = \max_{1 \leq i \leq n} \{D_n^+, D_n^-\}$$

The distribution of the K-S statistic does not depend on F_0 as long as F_0 is continuous.

To study the goodness-of-fit of the Gompertz model, we compute the Kolmogorov-Smirnov statistic between the empirical distribution function and the fitted distribution function when the parameters are obtained by method of maximum likelihood. We shall use the *ks.Gompertz()* function in R to perform the test. Now, we plot the empirical distribution function and the fitted distribution function using

proposed data sets in Figure 3-8 and the result of K-S test is shown in Table II.

TABLE II. D AND ITS CORRESPONDING P-VALUE USING KS-TEST

Data Set	D - value	P - value
1	0.062	0.8756
2	0.1759	0.2319
3	0.1553	0.6363
4	0.0962	0.3128
5	0.1694	0.1135
6	0.1267	0.8658

and ,

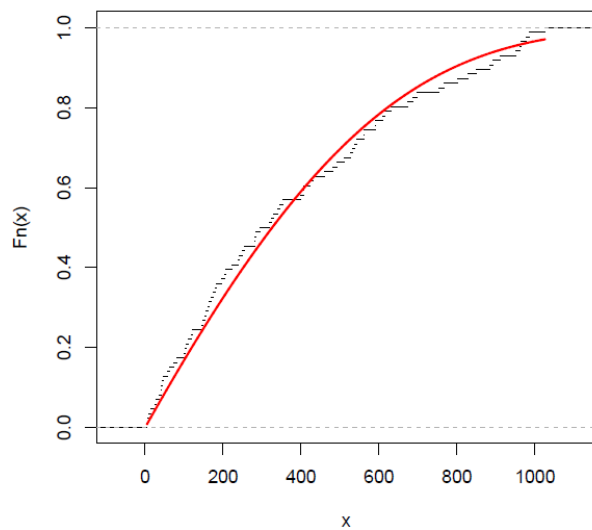


Figure 3. The graph for empirical distribution function and fitted distribution function for data set-1.

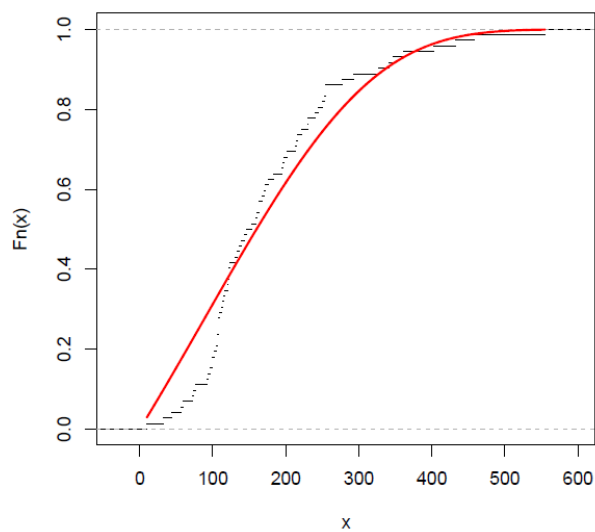


Figure 4. The graph for empirical distribution function and fitted distribution function for data set-2.

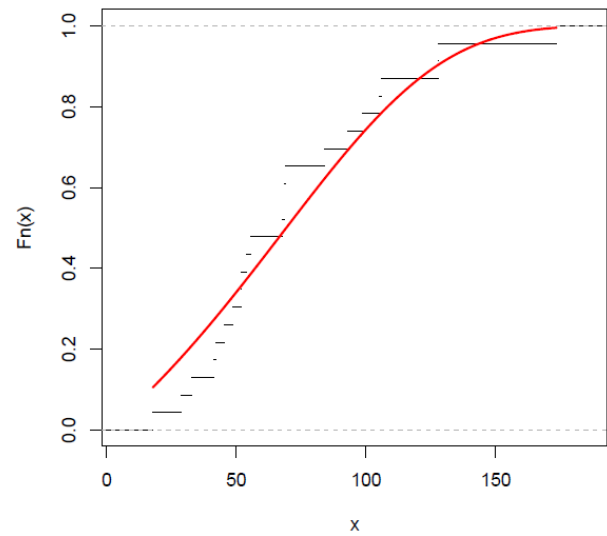


Figure 5. The graph for empirical distribution function and fitted distribution function for data set-3.

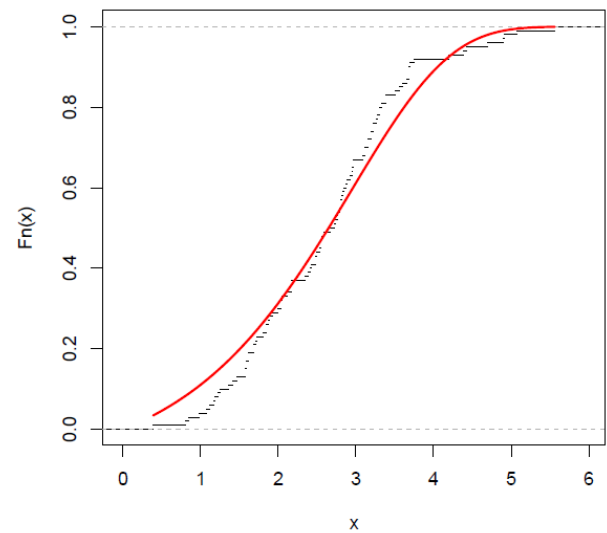


Figure 6. The graph for empirical distribution function and fitted distribution function for data set-4.

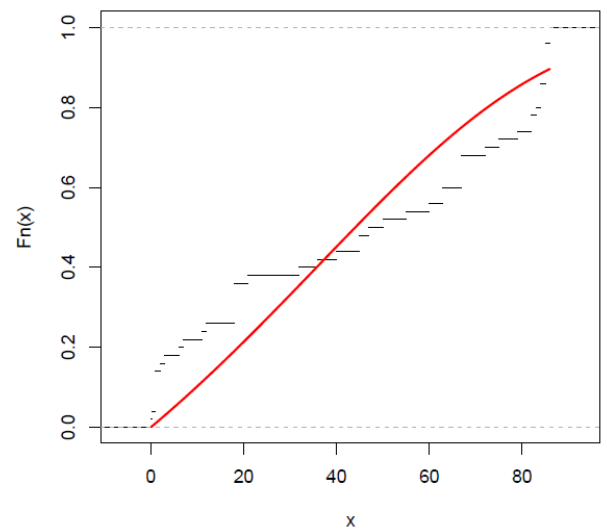


Figure 7. The graph for empirical distribution function and fitted distribution function for data set-5.

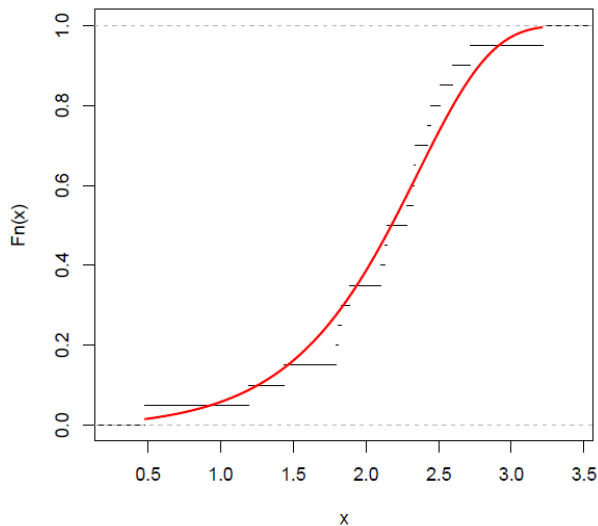


Figure 8. The graph for empirical distribution function and fitted distribution function for data set-6.

Since, the high p-value clearly indicates that those data set can be used to analyze Gompertz model and in this analysis from Table II data set-1, data set-6 and data set-3 having high p-value. Therefore from above result and Figure 3-8, it may clear that the estimated Gompertz model provides excellent good fit to the given data set-1, data set-6 and data set-3.

3.2 The Q-Q Plots Test

The Q-Q plot test is used to investigate whether an assumed model adequately fits a set of data. It helps the analyst to assess how well a given theoretical distribution fits the data.

Let $\hat{F}(x)$ be an estimate of $F(x)$ based on $x_1, x_2, \dots, x_n$.

The scatter plot of the points

$$\hat{F}^{-1}(p_{1:n}) \text{ versus } x_{1:n}, \quad i = 1, 2, \dots, n,$$

is called a Q-Q plot.

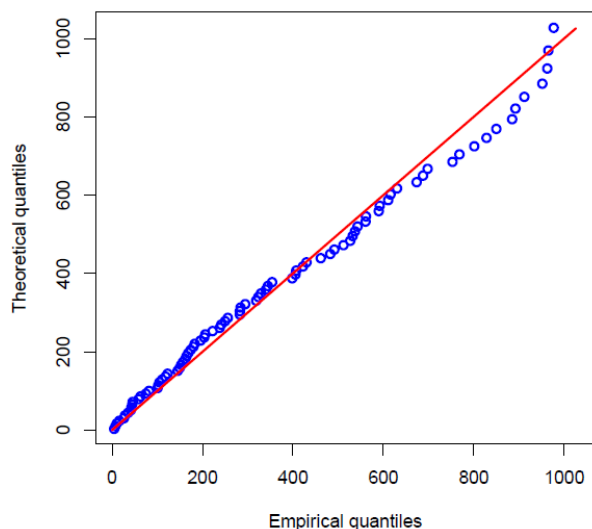


Figure 9. Quantile-Quantile(Q-Q) plot using MLEs as estimate for data set-1.

Thus, the Q-Q plots show the estimated versus the observed quantiles. If the model fits the data well, the pattern of points on the Q-Q plot will exhibit a 45-degree straight line. Note that all the points of a Q-Q plot are inside the square

$$[\hat{F}^{-1}(p_{1:n}), \hat{F}^{-1}(p_{n:n})] \times [x_{1:n}, x_{n:n}].$$

We shall use the *qq.Gompertz()* function to perform the test. We draw Quantile-Quantile (Q-Q) plot using MLEs as estimate for using different data set in given Figure 9-14

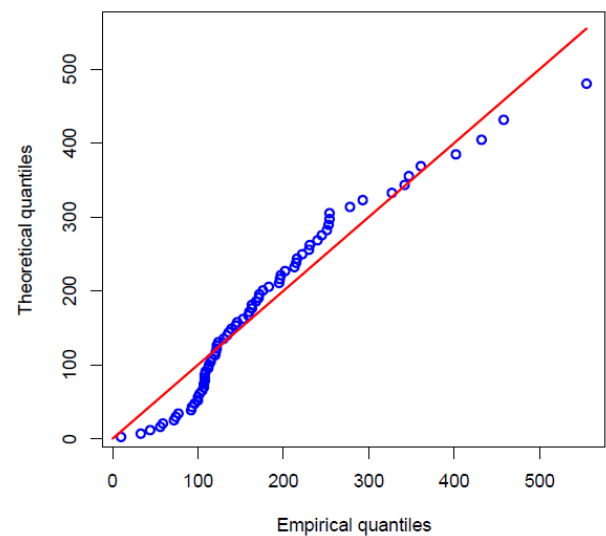


Figure 10. Quantile-Quantile(Q-Q) plot using MLEs as estimate for data set-2.

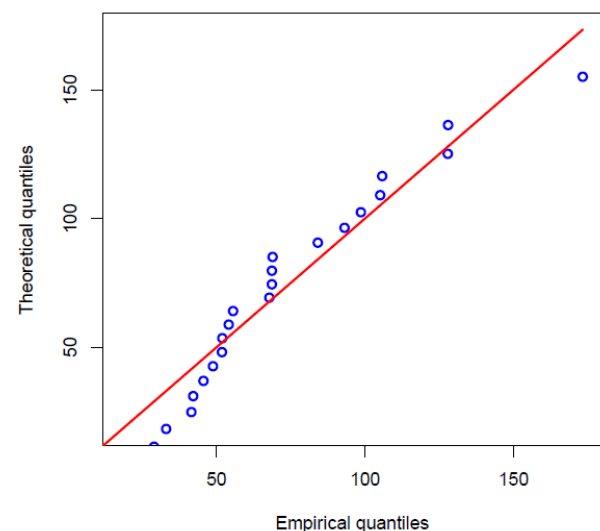


Figure 11. Quantile-Quantile(Q-Q) plot using MLEs as estimate for data set-3.

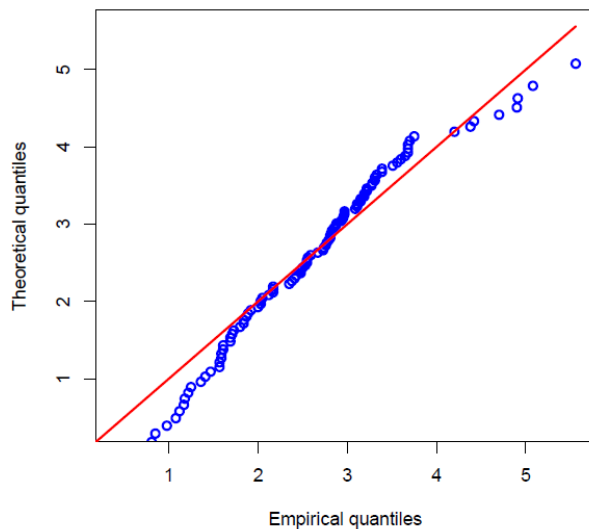


Figure 12. Quantile-Quantile(Q-Q) plot using MLEs as estimate for data set-4.

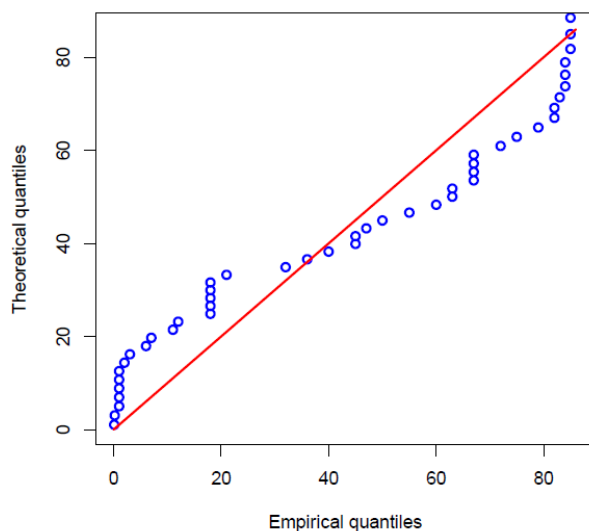


Figure 13. Quantile-Quantile(Q-Q) plot using MLEs as estimate for data set-5.

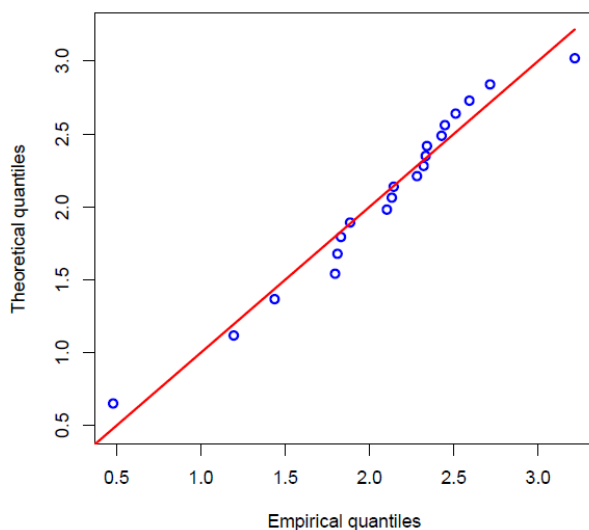


Figure 14. Quantile-Quantile(Q-Q) plot using MLEs as estimate for data set-6.

Thus, as can be seen from the straight line pattern in Figures 9-14, the Gompertz model much fits the data very well for data set-1.

4. CONCLUSION

An attempt has been made to incorporate Gompertz model for software reliability data. We have presented the statistical tools for empirical modeling of the data in general. These tools are developed in R language and environment for model analysis, model validation and estimation of parameters using method of maximum likelihood. To check the validity of the model, we have plotted a graph of empirical distribution function and fitted distribution function using KS-test for different data set and also we have to present power comparison between p-values of these data sets obtaining by K-S test for receiving feasible real data sets which are excellent good fit for analysis of Gompertz model. We have also discussed the Q-Q plots for model validation. Thus, from both used techniques of model validation for Gompertz model on different data set, the Gompertz model may fit the data very well only for data set-1, data set-6 and data set-3.

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